Parity Nonconservation from Division Algebras

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A novel origin of parity nonconservation is proffered within the context of the multigenerational lepto-quark model based upon the Division Algebras.

The notation and mathematical origins of this paper can be found in the references.

In **[1]** I extended the ideas of **[2]**, expanding a model with a single lepto-quark generation based on a 10-dimensional spacetime to one with three generations based on mixed copies of 14-dimensional spacetimes, and simultaneously obtaining internal symmetries $SU(2)$ and $SU(3)$ from the symmetries of the higher dimensions.

At the 10-dimensional level the derived $SU(2)$ generators were

$$\alpha q_{R1} = \begin{bmatrix} q_{R1} & 0 \\ 0 & -q_{R1} \end{bmatrix}, \ \alpha q_{R2} = \begin{bmatrix} q_{R2} & 0 \\ 0 & -q_{R2} \end{bmatrix}, \ \epsilon q_{R3} = \begin{bmatrix} q_{R1} & 0 \\ 0 & q_{R1} \end{bmatrix}.$$

Unlike the $SU(2)$ generators in **[3]**, these generators act on matter and antimatter the same. And neither this, nor that earlier $SU(2)$, violates parity (in **[3]** parity nonconservation arose from an analytical trick, which we dispense with here), although these generators do act differently on the left and righthand portions of the various spinors:

$$\Psi = \begin{bmatrix} \psi_l \\ \psi_r \end{bmatrix} = \frac{1}{2}(1+\alpha)\Psi + \frac{1}{2}(1-\alpha)\Psi.$$

The generators of the $SU(2)$ derived earlier were (rotated to fit the representation in **[1]**)

$$ie_{L7}q_{R1}, \ ie_{L7}q_{R2}, \ q_{R3}.$$

These generators act on left and righthand spinors the same, but differently on matter and antimatter:

$$\Psi = \rho_{L+}\Psi + \rho_{L-}\Psi = \frac{1}{2}(1+ie_{L7})\Psi + \frac{1}{2}(1-ie_{L7})\Psi.$$

(recall that $ie_{L7}\rho_{L\pm} = \pm\rho_{\pm}$).

The total Lie algebra generated by all these elements is $su(2) \times su(2)$, but we'll restrict our attention to the sums of these two sets of generators for the nonce. That is, the 3 elements:

$$\frac{1}{2}(\alpha q_{R1} + ie_{L7}q_{R1}\epsilon) = q_{R1}\begin{bmatrix} \rho_{L+} & 0 \\ 0 & -\rho_{L-} \end{bmatrix},$$

$$\frac{1}{2}(\alpha q_{R2} + ie_{L7}q_{R2}\epsilon) = q_{R2}\begin{bmatrix} \rho_{L+} & 0 \\ 0 & -\rho_{L-} \end{bmatrix},$$

$$\frac{1}{2}(\epsilon q_{R3} + q_{R3}\epsilon) = q_{R3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = q_{R3}\begin{bmatrix} \rho_{L+} & 0 \\ 0 & \rho_{L-} \end{bmatrix} + q_{R3}\begin{bmatrix} \rho_{L-} & 0 \\ 0 & \rho_{L+} \end{bmatrix}$$

(recall that $\rho_{L+} + \rho_{L-} = 1$).

We see that because $\rho_{L\pm}\rho_{L\pm} = \rho_{\pm}$ and $\rho_{L\pm}\rho_{L\mp} = 0$, the q_{R3} term breaks into a piece that anticommutes with the q_{R1} and q_{R2} terms, and a part that commutes, and so upon exponentiation will give rise to a separate $U(1)$. In particular,

$$U = exp(xq_{R1}\begin{bmatrix} \rho_{L+} & 0 \\ 0 & -\rho_{L-} \end{bmatrix} + yq_{R2}\begin{bmatrix} \rho_{L+} & 0 \\ 0 & -\rho_{L-} \end{bmatrix} + zq_{R3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})$$
$$= \begin{bmatrix} e^{u}\rho_{L+} & 0 \\ 0 & e^{\overline{u}}\rho_{L-} \end{bmatrix} + \begin{bmatrix} e^{zq_{R3}}\rho_{L-} & 0 \\ 0 & e^{zq_{R3}}\rho_{L+} \end{bmatrix}.$$

The action of this exponential on our spinor (here split into matter and anti-matter parts)

$$\Psi = \begin{bmatrix} \psi_l \\ \psi_r \end{bmatrix} = \begin{bmatrix} \rho_{L+}\psi_l \\ \rho_{L+}\psi_r \end{bmatrix} + \begin{bmatrix} \rho_{L-}\psi_l \\ \rho_{L-}\psi_r \end{bmatrix}$$

is

$$U\Psi = \begin{bmatrix} e^{u}\rho_{L+}\psi_l \\ e^{zq_{R3}}\rho_{L+}\psi_r \end{bmatrix} + \begin{bmatrix} e^{zq_{R3}}\rho_{L-}\psi_l \\ e^{\overline{u}}\rho_{L-}\psi_r \end{bmatrix},$$

where $u = xq_{R1} + yq_{R2} + zq_{R3}$, and $\overline{u} = -xq_{R1} + -yq_{R2} + zq_{R3}$, an automorphic conjugation of u. We see that lefthanded matter receives the full $SU(2)$, as does righthanded antimatter, while righthanded matter and lefthanded antimatter are only acted upon by $U(1)$. That's parity nonconservation without analytical tricks.

The results of this and my previous papers can be duplicated using matrices, ie., without any recourse to division algebras beyond **C**. However, that would rather defeat the purpose of this work. The results fall naturally into place when the division algebras are exploited, and their use frequently points to new avenues to explore. As to that, one final comment: this paper and its predecessor **[1]** were based on the expanded spinor space

$$\mathbf{C} \otimes \mathbf{H}^2 \otimes \mathbf{O}^3 = \mathbf{T}^6.$$

With another lifetime or two it would be interesting as a consequence to see if there is some new physics hiding in the lattice

$$\Lambda_2 \times \Lambda_8 \times \Lambda_{24}.$$

Certainly if one were inclined to think of our universe as discrete instead of continuous, this lattice offers an interesting starting point, incorporating what are arguably the three most important lattices in that area of mathematics.

References:

[1] G.M. Dixon, www.7stones.com/Homepage/6x6.pdf

[2] G.M. Dixon, www.7stones.com/Homepage/10Dnew.pdf

[3] G.M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers, and the Algebraic Design of Physics*, (Kluwer, 1994).