Division Algebras: InterFamily Mixing (Including Neutrinos)

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It is shown how a 14-dimensional spacetime derived from the Division Algebras mixes leptoquark families due to the breaking of isospin $SU(2)$.

In [1] the ideas in [2,3] were extended. A one family leptoquark model founded on the Division Algebras was extended to a three family model by expanding the spinor space in which these fermions reside from $\mathbf{C}\otimes\mathbf{H}^2\otimes\mathbf{O} = \mathbf{T}^2$ to

$$\mathbf{C}\otimes\mathbf{H}^2\otimes\mathbf{O}^3 = \mathbf{T}^6,$$

which is acted upon by $\mathbf{T}_A(6)$ (see references for notation). $\mathbf{T}_A(6)$ is not a Clifford algebra, but contains three copies of $\mathcal{CL}(1,13)$ (the Clifford algebra of 1,13-spacetime) which connect the families of the model in pairs.

Here we'll continue the ideas started in [1], with a focus on fermion interactions implied by a simple Dirac-like Lagrangian of the form

$$\mathcal{L} = \overline{\Psi}\ \partial\!\!\!/\Psi,$$

where Ψ resides in $\mathbf{T}^4$, and $\partial\!\!\!/$ is the Dirac operator for $\mathcal{CL}(1,13)\ equiv\mathbf{T}_A(4)$.

First some notation: we use the 2×2 real matrices

$$\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \alpha = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \gamma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

Define, for example, the following 4×4 real matrix:

$$[\beta\otimes\alpha] = \begin{bmatrix} 0 & \alpha \\ \alpha & 0 \end{bmatrix}.$$

Our $\mathcal{CL}(1,13)$ 1-vector basis consists of the following 14 elements:

1,3-spacetime:
$[\epsilon\otimes\beta](iq_{R3}),\ \ [\epsilon\otimes\gamma]q_{Lk}e_{L7}(iq_{R3}),\ k=1,2,3,$

6 dimensional space; $su(3)$ charges; matter/antimatter mixing:
$[\epsilon\otimes\gamma]ie_{Lp}(iq_{R3}),\ p=1,...,6,$

4 dimensional space; $su(2)$ charges; interfamily mixing:
$[\beta\otimes\epsilon]q_{R1},\ [\beta\otimes\epsilon]q_{R2},\ [\beta\otimes\alpha]q_{R3},\ [\gamma\otimes\alpha].$

It was shown in [3] how in a 1-family model based on $\mathcal{CL}(1,9)$ a Dirac Lagrangian like that above gave rise to matter-antimatter mixing mediated by the extra 6 dimensions. (Note: particular fermions can be associated with particular bits of Ψ with the aid of projection operators; incoming fermions reside in Ψ, and outgoing in $\overline{\Psi}$; if a particular incoming/outgoing combination contributes a real part to $\mathcal{L}$, it is considered an observable interaction.) These extra dimensions also carried color $SU(3)$ charges, and by replacing the 6 real dimensions by 3 complex dimensions the offending terms of the Lagrangian disappeared (offending because we have not observed matter spontaneously transmuting into antimatter; disappeared because, for example, the complex partial $(\partial_x + i\partial_y)f(x+iy) = 0$ identically).

In the present case we have 4 dimensions beyond the original extra 6. These carry isospin $SU(2)$ charges and give rise to interfamily mixing. As we did in the 6-dimensional case, if we replace the extra 4 dimensions with 2 complex dimensions we can make these mixing terms go away. However, $SU(3)$ is exact, and $SU(2)$ is broken, and it may not be justified to apply this method to all terms.

Begin by defining a pair of projection operators (idempotents) in $\mathbf{T}_A(4)$:

$$\Lambda_\pm = \frac{1}{2}(1 \pm i q_{R3}), \quad \Gamma_\pm = \frac{1}{2}(1 \pm [\alpha \otimes \epsilon]).$$

The two subspinors $\Gamma_\pm \Psi$ correspond to separate families (each is a 1,9-Dirac hyperspinor containing the 1,3-Dirac spinors of a family and its antifamily). The $\Lambda_\pm$ are $SU(2)$ isospin projectors. The part of $\partial\!\!\!/$ linear in the partials of these last 4 dimensions can be rewritten with respect to these projection operators as follows:

$$\begin{aligned}
&([\beta \otimes \epsilon] q_{R1} \partial_{10} + [\beta \otimes \epsilon] q_{R2} \partial_{11} + [\beta \otimes \alpha] q_{R3} \partial_{12} + [\gamma \otimes \alpha] \partial_{13}) \textstyle\sum_{\pm\pm} (\Lambda_\pm \Gamma_\pm) \\
&= (q_{R1}[\beta \otimes \epsilon](\partial_{10} - i\partial_{11}) - i[\beta \otimes \alpha](\partial_{12} - i\partial_{13}))\Lambda_+ \Gamma_+ \\
&+ (q_{R1}[\beta \otimes \epsilon](\partial_{10} + i\partial_{11}) + i[\beta \otimes \alpha](\partial_{12} + i\partial_{13}))\Lambda_- \Gamma_+ \\
&+ (q_{R1}[\beta \otimes \epsilon](\partial_{10} - i\partial_{11}) - i[\beta \otimes \alpha](\partial_{12} + i\partial_{13}))\Lambda_+ \Gamma_- \\
&+ (q_{R1}[\beta \otimes \epsilon](\partial_{10} + i\partial_{11}) + i[\beta \otimes \alpha](\partial_{12} - i\partial_{13}))\Lambda_- \Gamma_-
\end{aligned}$$

Meanwhile our Lagrangian term,

$$\mathcal{L} = \overline{\Psi}\ \partial\!\!\!/ \Psi = \begin{bmatrix} \Psi^*_{1r} & \Psi^*_{1l} & \Psi^*_{2r} & \Psi^*_{2l} \end{bmatrix} \partial\!\!\!/_{1,13} \begin{bmatrix} \Psi_{1l} \\ \Psi_{1r} \\ \Psi_{2l} \\ \Psi_{2r} \end{bmatrix}$$

contains the following $\Lambda_+\Gamma_+$ part, linear in the partials of the extra 4 dimensions:

$$\mathcal{L}_{++} = \begin{bmatrix} \Psi^*_{1r} & \Psi^*_{1l} & \Psi^*_{2r} & \Psi^*_{2l} \end{bmatrix} \begin{bmatrix} (q_{R1}(\partial_{10} - i\partial_{11}) - i(\partial_{12} - i\partial_{13}))\Lambda_+ \Psi_{2l} \\ (q_{R1}(\partial_{10} - i\partial_{11}) + i(\partial_{12} - i\partial_{13}))\Lambda_+ \Psi_{2r} \\ 0 \\ 0 \end{bmatrix}$$

The subscripts 1 and 2 indicate different leptoquark families, while l and r the left and right Chiralities. The assumption we're making here is that as Ψ_{2l} is an $SU(2)$ doublet, its dependence on the extra 4 dimensions reduces to 2 complex dimensions in such a way that the partials in this Lagrangian term are identically zero. This leaves us with the term:

$$\Psi^*_{1l}(q_{R1}(\partial_{10} - i\partial_{11}) + i(\partial_{12} - i\partial_{13}))\Lambda_+ \Psi_{2r}.$$

Since Ψ_{2r} is not an $SU(2)$ doublet we might assume that this term survives the partial, and since this is now associated with the term Ψ^*_{1l}, we infer that RH members of family 2 mix via these extra dimensions with LH members of family 1. This would imply more than just neutrino oscillations: it would also imply interfamily mixing of charged leptons, and of quarks. It is unclear at present how probable any of these interactions might be. A single family/antifamily spinor is associated with 1,9-spacetime, 6 dimensions of which are tightly curled and linked to the exact internal symmetry, $U(1) \times SU(3)$. Adding a second family requires 4 more dimensions, also presumably curled, and linked to the broken symmetry, isospin $SU(2)$. (The third family makes things mathematically more complex [**1**].) A good many things are unclear at this point, but the structure is there to blow away much of the fog.

References:

[1] G.M. Dixon, www.7stones.com/Homepage/6x6.pdf

[2] G.M. Dixon, www.7stones.com/Homepage/10Dnew.pdf

[3] G.M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers, and the Algebraic Design of Physics*, (Kluwer, 1994).