Matter Universe: A Mathematical Solution

Geoffrey Dixon
gdixon@7stones.com

The Standard Model arises from $\mathbf{T} := \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$. After summarizing how this happens, a new - and somewhat obvious - interpretation of these mathematical underpinnings is presented which requires 1,3-dimensional spacetimes to be fundamentally matter, or anti-matter. Both are required, and linked by and extra six space dimensions which carry color charges.

Spinors from Division Algebras

In this article the spinor is at the root of everything. My notion of what a spinor is derives from Ian Porteous's book *Topological Geometry* [1] which I was directed to some 30 years ago. Ian presented a table of representations of universal Clifford algebras of p-time:q-space dimensional spacetimes in terms of the first three real normed division algebras: **R**, **C**, and **H** (the remaining division algebra, the octonions, **O**, will enter shortly). I will employ Ian's notation **K**(n) to be the algebra of n×n matrices over an algebra **K**.

Let $\mathcal{CL}(p,q)$ be the Clifford algebra of p,q-spacetime (actually, timespace), then any of Ian's representations can be derived from the sequences for p = 0,1,2,3,..., and q = 0 (line 1), and the sequence for p = 0 and q = 0,1,2,3,... (line 2),

$\mathbf{R}$	$\mathbf{R^2}$	$\mathbf{R(2)}$	$\mathbf{C(2)}$	$\mathbf{H(2)}$	$\mathbf{H^2(2)}$	$\mathbf{H(4)}$	$\mathbf{C(8)}$	$\mathbf{R(16)}$	$\mathbf{R^2(16)}$	...
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H^2}$	$\mathbf{H(2)}$	$\mathbf{C(4)}$	$\mathbf{R(8)}$	$\mathbf{R^2(8)}$	$\mathbf{R(16)}$	$\mathbf{C(16)}$	...

and the rule

$$\mathcal{CL}(p+1,q+1) = \mathcal{CL}(p,q) \otimes \mathbf{R}(\mathbf{2}).$$

(There is also a periodicity (Bott) of order 8 indicated in the rows above. Also, **R**(2), being isomorphic to $\mathcal{CL}(1,1)$, provides a great way of adding time and a transverse space dimension to a pure (longitudinal) space Clifford algebra.)

Some things to point out in particular:

• In each case we can find a set of p+q anticommuting elements of $\mathcal{CL}(p,q)$ (the 1-vectors) the squares of which are $\pm I$, with I the identity of $\mathcal{CL}(p,q)$ (p +, and q -);

• The product of these p+q anticommuting 1-vectors is not a real multiple of the identity (this is the 'universal' part of 'universal Clifford algebra');

• The spinor space of $\mathcal{CL}(p,q)$ is the obvious set of n×1 column matrices over **R**, **C**, or **H**, on which our respective representations of $\mathcal{CL}(p,q)$ would most naturally act via

left matrix multiplication;
• If the underlying division algebra is $\mathbf{H}$, then multiplication on the spinor space by elements of $\mathbf{H}$ from the right is an algebra of actions on that space that is not accounted for by the elements of $\mathcal{CL}(p,q)$, and so it is *internal* with respect to those *external* Clifford algebra actions (in the sense that isospin $SU(2)$ is an *internal* symmetry, and in what follows it is from this right action by $\mathbf{H}$ that isospin $SU(2)$ arises in the form of the subset of unit elements, which is multiplicatively closed);
• And finally, the set of elements generated by taking the commutators of pairs of 1-vectors is the set of 2-vectors, and with respect to the commutator product this set is isomorphic to the Lie algebra $so(p,q) \simeq spin(p,q)$.

One last thing to note: each of the n components of these n×1 spinor columns is an element of $\mathbf{R}$, $\mathbf{C}$, or $\mathbf{H}$, a division algebra. Conventionally $\mathcal{CL}(3,0)$ is represented by the algebra $\mathbf{C}(\mathbf{2})$, but

$$\mathbf{C}(\mathbf{2}) \simeq \mathbf{P} := \mathbf{C} \otimes \mathbf{H},$$

the complexified quaternions. These two versions of $\mathcal{CL}(3,0)$ have different spinor spaces. In the former case the spinor space is the 4-dimensional (over $\mathbf{R}$) and consisting of 2×1 complex matrices; and in the latter case the spinor space is the 8-dimensional 1-component set $\mathbf{C} \otimes \mathbf{H}$ itself. In this latter case the algebra of actions of $\mathbf{H}$ multiplication on a spinor from the right is again not accounted for and commutes with the Clifford algebra actions. In addition, in this case the single spinor component is not an element of a division algebra, but of $\mathbf{C} \otimes \mathbf{H}$, which has a nontrivial decomposition of its identity into a pair of mutually orthogonal idempotents that sum to 1. More on this kind of thing very soon: it is the key to almost everything.

Octonions as Spinors

$\mathbf{O}$ is an 8-dimensional real algebra, and despite its nonassociativity it can be incorporated into this Clifford algebra and spinor scheme. First some notation:

$$\forall\, x, w \in \mathbf{O},\ \ \mathrm{L}_x[w] = xw,\ \ \mathrm{R}_x[w] = wx.$$

However, although this notation is somewhat conventional, in all my previous work I've used the following notation (thereby avoiding subscripts on subscripts):

$$x_L \equiv \mathrm{L}_x,\ \ x_R \equiv \mathrm{R}_x.$$

In particular, I use a basis $e_a,\ a = 0, 1, ..., 7$, for $\mathbf{O}$, with $e_0 = 1$ the identity, and

$$\{e_0, e_{1+k}, e_{2+k}, e_{4+k}\}$$

is a basis for a quaternionic subalgebra for all integers k, where the index summation is modulo 7, from 1 to 7. Define

$$e_{Lab...c} \equiv \mathrm{L}_{e_a}\mathrm{L}_{e_b}...\mathrm{L}_{e_c},\ \ e_{Rab...c} \equiv \mathrm{R}_{e_c}...\mathrm{R}_{e_b}\mathrm{R}_{e_a},$$

(note reversal of indices in second case), and let $\mathbf{O}_L$ and $\mathbf{O}_R$ be the algebras spanned by these respective sets of left and right actions. Finally, and most importantly,

$$\mathbf{O}_L = \mathbf{O}_R \simeq \mathbf{R}(8),$$

so these are each the full algebras of endomorphisms on 8-dimensional $\mathbf{O}$. Any element of $\mathbf{O}_L$ can be expressed as a linear combination of elements of $\mathbf{O}_R$, and visa versa. More on this later.

So,

$$\mathbf{O}_L \simeq \mathbf{R}(8) \simeq \mathcal{CL}(0,6),$$

and the spinor space of this representation of $\mathcal{CL}(0,6)$ as $\mathbf{O}_L$ is just $\mathbf{O}$ itself, which, unlike $\mathbf{R}^8$, has a natural multiplicative structure. The spinor space itself is a division algebra. (Note: $\mathbf{O}_L$ is trivially associative.) We can represent a basis for the Clifford algebra 1-vectors in this case as

$$e_{Lp},\ p = 1,2,3,4,5,6.$$

The set of 2-vectors is then spanned by

$$e_{Lpq},\ p,q \in \{1,2,3,4,5,6\},\ p \neq q,$$

and given the commutator product this is the Lie algebra $so(6)$. The 6-vector is

$$\prod_{p=1}^{p=6} \mathrm{L}_{e_p} = \mathrm{L}_{e_7} = \prod_{p=1}^{p=6} e_{Lp} = e_{L7}.$$

This is how I write out the multiplication table whenever I need a quick reference:

$$\begin{array}{c} 124 \\ 235 \\ 346 \\ 457 \\ 561 \\ 672 \\ 713 \end{array}$$

These are the 7 sets of "quaternionic" index triples. For example, from this I deduce that $e_6 e_1 = -e_1 e_6 = e_5$. In general, if a and b are distinct indices from 1 to 7, then $e_a e_b$ will be equal to $\pm e_c$ for some other index c, the sign positive if $b - a$ is a power of 2, and negative otherwise ($b - a$ taken modulo 7, from 1 to 7, so 2-5 = 4, and therefore $e_5 e_2$ is positive (e_3)). Note: in [2] a dual to this multiplication table was used.

Complexified Octonions as Spinors

Define

$$\mathbf{S} = \mathbf{C} \otimes \mathbf{O}.$$

Since $\mathbf{O}_L = \mathbf{O}_R$, and trivially $\mathbf{C}_L = \mathbf{C}_R$, the algebra of left or right actions of $\mathbf{S}$ on itself is

$$\mathbf{S}_L = \mathbf{S}_R = \mathbf{C} \otimes \mathbf{O_L} \simeq \mathbf{C} \otimes \mathbf{R}(\mathbf{8}) = \mathbf{C}(\mathbf{8}) \simeq \mathcal{CL}(\mathbf{7}, \mathbf{0}).$$

Quickly then, so we can get to the res, $\mathbf{S}_L$ viewed as the Clifford algebra $\mathcal{CL}(7,0)$ has the following natural identifications:

1-vectors:		$ie_{La},\ a \in \{1,...,7\},$
2-vectors:	$(so(7))$	$e_{Lab},\ a,b \in \{1,...,7\}, a \neq b,$
...		
7-vector		$\prod_{a=1}^{a=7} i\mathrm{L}_{e_a} = -ie_{L1234567} = i.$

The spinor space in this case is $\mathbf{S}$ itself, and as was true of the previous case, this spinor space has an algebraic structure of its own. However, in the previous case the spinor space, $\mathbf{O}$, was a division algebra; $\mathbf{S}$ is not, and it admits a nontrivial resolution of its identity into a pair of orthogonal projectors (idempotents, as long as everything is alternative). These are

$$\rho_\pm = \frac{1}{2}(1 \pm ie_7)$$

(this selection is clearly not unique, but dates back almost 40 years in the literature [6], and is, given my choice of octonion multiplication table, rather natural).

The presence of these projectors means there is a natural (ok, I'm over-using that word) decomposition of the spinor space $\mathbf{S}$ into 4 mutually orthogonal subspaces:

$$\begin{aligned}
\mathbf{S}_{++} &= \rho_+ \mathbf{S} \rho_+ = \rho_{L+}\rho_{R+}[\mathbf{S}], \quad \text{1-d over } \mathbf{C} \\
\mathbf{S}_{+-} &= \rho_+ \mathbf{S} \rho_- = \rho_{L+}\rho_{R-}[\mathbf{S}], \quad \text{3-d over } \mathbf{C} \\
\mathbf{S}_{-+} &= \rho_- \mathbf{S} \rho_+ = \rho_{L-}\rho_{R+}[\mathbf{S}], \quad \text{3-d over } \mathbf{C} \\
\mathbf{S}_{--} &= \rho_- \mathbf{S} \rho_- = \rho_{L-}\rho_{R-}[\mathbf{S}], \quad \text{1-d over } \mathbf{C}
\end{aligned}$$

where $\rho_{L\pm} = \frac{1}{2}(1 \pm ie_{L7})$, and $\rho_{R\pm} = \frac{1}{2}(1 \pm ie_{R7})$, which provides an expression of this decomposition in terms of projectors in $\mathcal{CL}(7,0)$.

These four reductions of $\mathbf{S}$ into four orthogonal subspaces have corresponding reductions of $\mathcal{CL}(7,0)$ into subalgebras that map the four subspaces to themselves. These are:

$$\begin{aligned}
\mathcal{CL}(7,0) &\longrightarrow \rho_{L+}\rho_{R+}\mathcal{CL}(7,0)\rho_{L+}\rho_{R+} = \mathcal{CL}_\rho(7,0)\rho_{L+}\rho_{R+}, \\
\mathcal{CL}(7,0) &\longrightarrow \rho_{L+}\rho_{R-}\mathcal{CL}(7,0)\rho_{L+}\rho_{R-} = \mathcal{CL}_\rho(7,0)\rho_{L+}\rho_{R-}, \\
\mathcal{CL}(7,0) &\longrightarrow \rho_{L-}\rho_{R+}\mathcal{CL}(7,0)\rho_{L-}\rho_{R+} = \mathcal{CL}_\rho(7,0)\rho_{L-}\rho_{R+}, \\
\mathcal{CL}(7,0) &\longrightarrow \rho_{L-}\rho_{R-}\mathcal{CL}(7,0)\rho_{L-}\rho_{R-} = \mathcal{CL}_\rho(7,0)\rho_{L-}\rho_{R-},
\end{aligned}$$

where the subalgebra $\mathcal{CL}_\rho(7,0)$ is the same for all four reductions, so we will just look at the (++)-reduction. (Why are the $\mathcal{CL}_\rho(7,0)$ the same? In each case the reduction occurs when one of the ρ's goes through the Clifford algebra. If e_{L7} (e_{R7}) anticommutes

with a piece of $\mathcal{CL}(7,0)$, then $\rho_{L\pm}$ ($\rho_{R\pm}$) will change to $\rho_{L\mp}$ ($\rho_{R\mp}$) when drawn from one side of that piece to the other, and when it gets there it will encounter $\rho_{L\pm}$ ($\rho_{R\pm}$), and the resulting product is zero, so that piece will be "reduced" out. So the sign in $\rho_{L\pm}$ ($\rho_{R\pm}$) is immaterial.) Note first that

$$\begin{array}{lll}
\rho_{L\pm}\rho_{L\pm} = \rho_{L\pm} & \Rightarrow & \rho_{L\pm}e_{La}\rho_{L\pm} = e_{La}\rho_{L\pm}, a = 0, 7, \\
\rho_{L\pm}\rho_{L\mp} = 0 & \Rightarrow & \rho_{L\pm}e_{La}\rho_{L\pm} = e_{La}\rho_{L\mp}\rho_{L\pm} = 0, a = 1, ..., 6, \\
\rho_{R\pm}\rho_{R\pm} = \rho_{R\pm} & \Rightarrow & \rho_{R\pm}e_{Ra}\rho_{R\pm} = e_{Ra}\rho_{R\pm}, a = 0, 7, \\
\rho_{R\pm}\rho_{R\mp} = 0 & \Rightarrow & \rho_{R\pm}e_{Ra}\rho_{R\pm} = e_{Ra}\rho_{R\mp}\rho_{R\pm} = 0, a = 1, ..., 6.
\end{array}$$

Therefore, of the seven 1-vectors of $\mathcal{CL}(7,0)$, the only one that survives the reduction to $\mathcal{CL}_\rho(7,0)$ is ie_{L7}. (Oh, and by the way, $\rho_{L\pm}$ commutes with $\rho_{R\pm}$.)

However, what we're really interested in is what happens to the 2-vectors, viewed as a representation of the Lie algebra $so(7)$. There are 21 elements, $e_{Lab},\ a, b \in \{1, 2, 3, 4, 5, 6, 7\}$ distinct, but we can divide these into two types: those for which one of the indices is 7; and those for which neither index is 7. In what follows it will be understood that any index $p, q, r, s \in \{1, 2, 3, 4, 5, 6\}$. Ok, so

$$\begin{array}{l}
\rho_{L\pm}e_{Lp7}\rho_{L\pm} = \rho_{L\pm}e_{Lp}e_{L7}\rho_{L\pm} = e_{Lp}\rho_{L\mp}e_{L7}\rho_{L\pm} = e_{Lp}e_{L7}\rho_{L\mp}\rho_{L\pm} = 0, \\
\rho_{L\pm}e_{Lpq}\rho_{L\pm} = \rho_{L\pm}e_{Lp}e_{Lq}\rho_{L\pm} = e_{Lp}\rho_{L\mp}e_{Lq}\rho_{L\pm} = e_{Lp}e_{Lq}\rho_{L\pm}\rho_{L\pm} = e_{Lpq}\rho_{L\pm},
\end{array}$$

where the subalgebra of $so(7)$ generated by elements $e_{Lpq},\ p, q \in \{1, ..., 6\}$ distinct, is $so(6)$. But we're not quite done, since we still have to finish the reduction by looking at $\rho_{R\pm}e_{Lpq}\rho_{L\pm}\rho_{R\pm}$. Since the $\rho_{L\pm}$ is irrelevant, we'll leave it out for now and just look at the elements $\rho_{R\pm}e_{Lpq}\rho_{R\pm}$.

Once again we're going to divide these 15 index combinations into 2 sets: those for which $e_pe_q = \pm e_7$; and those for which $e_pe_q = \pm e_r,\ r \neq 7$. We'll consider the latter case first, and there are 12 distinct elements (to within a sign). We need only look at one, which will be e_{L12} (note: $e_1e_2 = e_4$). Recall, $\mathbf{O}_L = \mathbf{O}_R$, so we can express any element of $\mathbf{O}_L$ as a linear combination of elements of $\mathbf{O}_R$. In particular, given the multiplication table employed here [2]:

$$e_{L12} = \frac{1}{2}(e_{R4} - e_{R12} + e_{R63} + e_{R57}).$$

Therefore,

$$\begin{array}{ll}
\rho_{R\pm}e_{L12}\rho_{R\pm} & = \frac{1}{2}\rho_{R\pm}(e_{R4} - e_{R12} + e_{R63} + e_{R57})\rho_{R\pm} \\
& = \frac{1}{2}(\rho_{R\pm}\rho_{R\mp}e_{R4} - \rho_{R\pm}\rho_{R\pm}e_{R12} + \rho_{R\pm}\rho_{R\pm}e_{R63} + \rho_{R\pm}\rho_{R\mp}e_{R57}) \\
& = \frac{1}{2}\rho_{R\pm}(-e_{R12} + e_{R63}) \\
& = \frac{1}{2}(e_{L12} - e_{L63})\rho_{R\pm}.
\end{array}$$

Note: $\rho_{R\pm}$ commutes with $(e_{L12} - e_{L63}) = (-e_{R12} + e_{R63})$. The other eleven e_{Lpq}, such that $e_pe_q = \pm e_r,\ r \neq 7$, reduce in like fashion when surrounded with $\rho_{R\pm}$, and these 12 elements are not linearly independent. For example,

$$\rho_{R\pm}e_{L63}\rho_{R\pm} = -\frac{1}{2}(e_{L12} - e_{L63})\rho_{R\pm}.$$

So, in fact there are only 6 independent elements surviving the reduction ($\rho_{R\pm}...\rho_{R\pm}$) of these 12 elements. These are 6 of the 8 elements of the $su(3)$ Lie algebra that generate an $SU(3)$ subgroup of the Lie group G_2, the automorphism group of $\mathbf{O}$, that leave the unit $e_7 \in \mathbf{O}$ invariant.

The final 3 elements of $so(6)$ we need look at are the e_{Lpq} for which $e_p e_q = \pm e_7$. These are

$$\begin{aligned} e_{L13} &= \tfrac{1}{2}(e_{R7} - e_{R13} + e_{R26} + e_{R45}), \\ e_{L26} &= \tfrac{1}{2}(e_{R7} + e_{R13} - e_{R26} + e_{R45}), \\ e_{L45} &= \tfrac{1}{2}(e_{R7} + e_{R13} + e_{R26} - e_{R45}), \end{aligned}$$

and (I hope this is obvious) $\rho_{R\pm}$ commutes with every term on the right hand side of these equations, so there is no further reduction achieved at this point. However, we can take linear combinations of these 3 elements to make it clearer what the overall structure of $\rho_{R\pm} so(6) \rho_{R\pm}$ actually is. In particular, there are 2 linearly independent elements we get by taking the differences of these 3 in pairs. Together with the 6 elements we got above, we now have a complete basis for $su(3) = \text{span}\{(e_{Lpq} - e_{Lrs})\}$, where $p, q, r, s \in \{1, 2, 3, 4, 5, 6\}$, and $e_p e_q = e_r e_s$. Again, this generates the $SU(3)$ subgroup of G_2 that leaves e_7 invariant.

The final element we get by taking the sum of the 3 elements above. In particular, let

$$\mu = \frac{1}{6}(e_{L13} + e_{L26} + e_{L45}) = \frac{1}{6}(e_{L7} - e_{L7} + e_{L13} + e_{L26} + e_{L45}) = \frac{1}{6}(e_{L7} + 2e_{R7}).$$

This element commutes with the elements of $su(3)$, and together they constitute a $u(3)$ subalgebra of $so(6)$, which is a subalgebra of our initial $so(7)$.

In fact, however, we have four variations on this full reduction, each acting nontrivially on only one of the four subspaces of our spinor space $\mathbf{S}$.

$$\begin{aligned} u(3)\rho_{L+}\rho_{R+} &: \mathbf{S}_{++} \quad (su(3) \text{ singlet}), \\ u(3)\rho_{L+}\rho_{R-} &: \mathbf{S}_{+-} \quad (su(3) \text{ triplet}), \\ u(3)\rho_{L-}\rho_{R+} &: \mathbf{S}_{-+} \quad (su(3) \text{ antitriplet}), \\ u(3)\rho_{L-}\rho_{R-} &: \mathbf{S}_{--} \quad (su(3) \text{ antisinglet}). \end{aligned}$$

And as to μ, it has the following actions on the four subspaces:

$$\begin{aligned} \mu\mathbf{S}_{++} &= -\tfrac{i}{2}\mathbf{S}_{++}, \\ \mu\mathbf{S}_{+-} &= +\tfrac{i}{6}\mathbf{S}_{+-}, \\ \mu\mathbf{S}_{-+} &= -\tfrac{i}{6}\mathbf{S}_{-+}, \\ \mu\mathbf{S}_{--} &= +\tfrac{i}{2}\mathbf{S}_{--}. \end{aligned}$$

Anyone familiar with the Standard Model of quarks and leptons will recognize this as the $u(1)$ hypercharge generator, or what can be interpreted as such. To this point it's just pure mathematics.

$\mathbf{T}^2$ Spinors: Particle Identifications and the New Interpretation

Bases for the real division algebras, $\mathbf{C}$, $\mathbf{H}$, $\mathbf{O}$ (complex algebra, quaternions, and octonions), are [2][3][5]:

$$\begin{array}{ll} \mathbf{C} & \{1,\ i\} \\ \mathbf{H} & \{q_0 = 1,\ q_k,\ k = 1, 2, 3\} \\ \mathbf{O} & \{e_0 = 1,\ e_a,\ a = 1, ..., 7\} \end{array}$$

The algebra

$$\mathbf{T} := \mathbf{C} \otimes \mathbf{H} \otimes \mathbf{O}$$

is $2 \times 4 \times 8 = 64$-dimensional. It is noncommutative, nonassociative, and nonalternative.

Although I consider it but a restricted model of reality, the basis of what I will do here is the 10-dimensional space-time model developed in [2], with mathematical expansion to be found in [3]. In this model, which accounts for a single family of quarks and leptons, and a corresponding antifamily, the foundation is the 128-dimensional hyperspinor space

$$\mathbf{T}^2$$

(the doubling of $\mathbf{T}$ in the spinor space is modeled on the notion that a Dirac spinor is a double Pauli spinor).

A Dirac spinor is acted upon by the Dirac algebra,

$$\mathbf{C}(4).$$

But I use a division algebra representation of the Pauli algebra,

$$\mathbf{P}_L := \mathbf{C} \otimes \mathbf{H}_L,$$

and in this case the Dirac algebra is

$$\mathbf{P}_L(2).$$

This is the complexification of the Clifford algebra of 1,3-spacetime. Likewise $\mathbf{T}^2$ is acted upon by the complexification of the Clifford algebra of 1,9-spacetime, represented by

$$\mathbf{T}_L(2),$$

where $\mathbf{T}_L$ is the algebra of left actions of $\mathbf{T}$ on itself, which in the octonion case, due to nonassociativity, requires the nesting of actions.

If memory serves (and it serves less well every year), von Neumann and others [7], in their efforts to expand quantum theory from a foundation on $\mathbf{C}$ to one on $\mathbf{O}$, linked quantum observability with algebraic associativity, and unobservability with nonassociativity, thinking along these lines being forced by the nonassociativity of $\mathbf{O}$. It was this recollection, although not entirely relevant, as their work revolved around using $\mathbf{O}$ as a foundation for quantum theory, that inspired what follows.

In the Standard Model, or at least in the real world, quarks are not observable. In the **T**-theory developed in [2][3], and elsewhere, the quarks are associated with the octonion units, $e_p, p = 1, ..., 6$. The extra six space dimensions are also rest on these units, and they too are evidently not directly observable.

My model building in [2][3] relies heavily on the resolution of the identity of

$$\mathbf{S} := \mathbf{C} \otimes \mathbf{O}$$

into a pair of orthogonal idempotents,

$$\rho_{\pm} = \frac{1}{2}(1 \pm ie_7)$$

discussed above. With these $\mathbf{S}$ was divided into 4 orthogonal subspaces:

$$\begin{array}{ll} \mathbf{S}_{++} & = \rho_+ \mathbf{S} \rho_+, \\ \mathbf{S}_{--} & = \rho_- \mathbf{S} \rho_-, \\ \mathbf{S}_{+-} & = \rho_+ \mathbf{S} \rho_-, \\ \mathbf{S}_{-+} & = \rho_- \mathbf{S} \rho_+. \end{array}$$

Both $\mathbf{S}_{++}$ and $\mathbf{S}_{--}$ are associative subalgebras of $\mathbf{S}$ isomorphic to $\mathbf{C}$. $\mathbf{S}_{+-}$ and $\mathbf{S}_{-+}$ are not subalgebras, but they are highly nonassociative (this nonassociativity implying $\mathbf{S}^2_{\pm\mp} = \mathbf{S}_{\mp\pm}$). Elements of the first two sets are linear (over $\mathbf{C}$) in the octonions $\{e_0 = 1,\ e_7\}$ (lepton and anti-lepton parts), and the second two sets linear over $\{e_p,\ p = 1, 2, 3, 4, 5, 6\}$ (quark and anti-quark).

An elegant representation of the Clifford algebra $\mathcal{CL}(1, 9)$ represented in $\mathbf{T}_L(2)$ that is aligned with the choice of the octonion unit e_7 to appear in $\rho_{\pm}$ arises from the following set of ten anti-commuting 1-vectors:

$$\beta,\ \ \gamma q_{Lk} e_{L7},\, k = 1, 2, 3,\ \ \gamma i e_{Lp},\, p = 1, ..., 6,$$

where

$$\epsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \alpha = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},\ \beta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},\ \gamma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

and as usual the subscripts "L" and "R" signify an action from the left or the right on $\mathbf{T}$. (So, for example,

$$\mathbf{S}_{+-} = \rho_+ \mathbf{S} \rho_- = \rho_{L+} \rho_{R-}[\mathbf{S}].)$$

Our observable spacetime has 3 space dimensions, not 9. There are two (what I would call) canonical ways of reducing the 1-vectors of $\mathcal{CL}(1, 9)$, a mix of observable and unobservable dimensions, to the 1-vectors of observable $\mathcal{CL}(1, 3)$:

$$\rho_{L\pm}\ \{\beta,\ \ \gamma q_{Lk} e_{L7},\, k = 1, 2, 3,\ \ \gamma i e_{Lp},\, p = 1, ..., 6\} \rho_{L\pm}$$

$$= \ \{\beta,\ \ \gamma i q_{Lk},\, k = 1, 2, 3\} \rho_{L\pm}.$$

These two collections of $\mathcal{CL}(1,3)$ 1-vectors act on half of the full spinor space $\mathbf{T}^2$. In particular, they act respectively on

$$\rho_{L\pm}[\mathbf{T}^2] = \rho_{\pm}\mathbf{T}^2,$$

where the underlying mathematics implies that these are, respectively, the matter and anti-matter halves of $\mathbf{T}^2$ ($\rho_+\mathbf{T}^2$ being a full family of lepton and quark Dirac spinors, and $\rho_-\mathbf{T}^2$ the corresponding anti-family) (see [2][3]).

Our observable universe is a 1,3-spacetime. There are those two ways of reducing the initial 1,9-spacetime above to 1,3-spacetimes, one associated with matter, one anti-matter. It now seems perfectly obvious to me to interpret this to mean that our observable 1,3-spacetime must be one, or the other (since it is ours, we call it matter). That is, the observable (habitable, if you will) spacetime in which we reside must of necessity be a matter universe, with anti-matter arising from secondary interactions - or an anti-matter universe - and that both must exist. The question therefore arises, if there is just our matter universe, and a single separate anti-matter universe, do they evolve in tandem? That is, is there an anti-matter me presently typing an anti-matter version of this article on an anti-computer? Curiously, that anti-me doubtless thinks of himself as being composed of matter. He is wrong, of course. I am the matter me; he the anti-matter.

However, this tandem evolution seems unlikely, for these matter and anti-matter 1,3-spacetimes, although distinct, can interact with each other through the remaining 6 dimensions of our initial 1,9-dimensional spacetime. Quarks, like these extra 6 dimensions of space in this model, are evidently not directly observable. And like the extra 6 dimensions of space, they owe their existence to the octonion units $e_p,\ p = 1,2,3,4,5,6$. To reduce the spinor space $\mathbf{T}^2$ all the way to its observable lepton part (the anti-lepton part is similar) we need an extra ρ_+. Specifically,

$$\rho_{L\pm}\rho_{R\pm}[\mathbf{T}^2] = \rho_+\mathbf{T}^2\rho_+$$

is a lepton doublet, consisting of 2 Dirac spinors, one for the electron, one for its neutrino. (The particle identifications are not arbitrary. See particularly [3] for the mathematics behind that statement.) Interestingly, this further reduction does not result in any further reduction of the 1-vector space of our original Clifford algebra, $\mathcal{CL}(1,9)$. We're still left with a version of 1-vectors for $\mathcal{CL}(1,3)$. However, the story is different for the space of 2-vectors, which we saw above in the case of $\mathbf{S}$. Initially they form a representation of the 1,9-Lorentz Lie algebra, $so(1,9)$. After the initial reduction we get something more than $so(1,3)$:

$$\rho_{L+}so(1,9)\rho_{L+} = (so(1,3) \times so(6))\rho_{L+},$$

and after the second spinor reduction,

$$\rho_{R+}\rho_{L+}so(1,9)\rho_{L+}\rho_{R+} = (so(1,3) \times u(1) \times su(3))\rho_{L+}\rho_{R+}.$$

This is precisely what it seems.

The situation is more complicated than this (see [2][3]), but the overriding point being made here is that the mathematics of $\mathbf{T}$ can be viewed as implying we exist in an observable universe that must be dominantly matter, or anti-matter (if we accept that everything carrying nontrivial $SU(3)$ color charges is not directly observable by us, which in this context includes quarks, anti-quarks, and the extra 6 dimensions of spacetime, all of which involve the octonion units, $e_p,\ p = 1, 2, 3, 4, 5, 6$, which carry those charges). Acceptance of this notion has the potential to imply far more profound things about physics.

For example, in [2] it was pointed out that the original model allowed algebraically for matter-antimatter mixing via the extra 6 dimensions, but that reasonable conditions put on the dependence of the various particle fields on these extra dimensions led to these mixing pathways disappearing. Whatever the case, in the present context this idea of mixing needs to be rethought. The extra 6 dimensions provide channels from the matter universe to the antimatter universe. Were these channels viable they might allow, for example, an electron from our matter universe to channel through to the anitmatter universe, appearing on the other side as an antiquark (it necessarily picks up an anti-color charge en route). At the very least they would allow the particles of our matter spacetime to influence, and be influenced by, the particles of the anti-matter spacetime. And this just scratches the surface.

A final comment: this exploitation of $\mathbf{T}$ as the foundation of a model of reality is not the only one, it is the one I like best (well, I've been at it for over 30 years, so changing now is not going to happen). For an alternate approach, see [8].

References

[1] I.R. Porteous, *Topological Geometry*, (Cambridge, 2nd Ed., 1981).

[2] G.M. Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers, and the Algebraic Design of Physics*, (Kluwer (Now Springer), 1994).

[3] G.M. Dixon, *Division Algebras, Lattices, Physics, Windmill Tilting*, (CreateSpace, 2011).

[4] G.M. Dixon, J. Math. Phys. 45, 3878 (2004).

[5] http://www.7stones.com

[6] M. Günaydin, F. Gürsey, Quark Structure and the Octonions, J. Math. Phys., Vol. 14, No. 11 (1973).

[7] Pascual Jordan, John von Neumann, Eugene Wigner, *On an algebraic generalization of the quantum mechanical formalism*, Ann. Math. 35 (1934), 29-64.

[8] Cohl Furey, *Unified theory of ideals*, Phys. Rev. D 86 (2012).

[9] John Baez, The octonions, Bull. Amer. Math. Soc. 39 (2002), 145-205